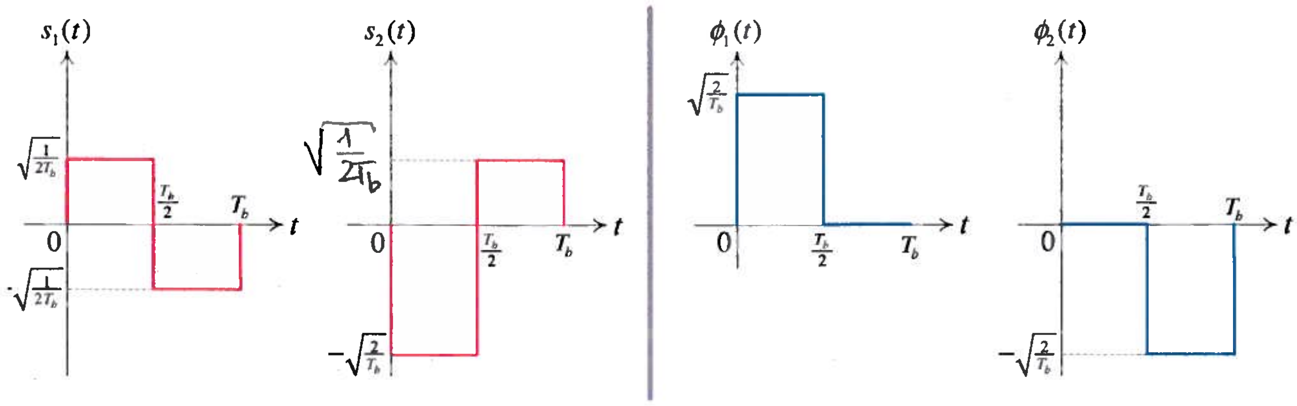
Consider a binary signal set $\{s_1(t), s_2(t)\}$ shown below. It is also known that the two orthonormal basis functions $\{\phi_1(t), \phi_2(t)\}$ shown on the figure can be used to represent the signal set *exactly*.




[2] (a) **Compute the energies, $E_1$ and $E_2$, of the two signals.**

$$E_1 = \int_0^{T_b} s_1^2(t)\,dt = \frac{1}{2T_b} \cdot T_b = \frac{1}{2}$$

$$E_2 = \int_0^{T_b} s_2^2(t)\,dt = \frac{2}{T_b} \cdot \frac{T_b}{2} + \frac{1}{2T_b} \cdot \frac{T_b}{2} = \frac{5}{4}$$

[4] (b) **Determine the coefficients $s_{ij}$ in the representation $s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t)$, $i, j \in \{1, 2\}$. Then clearly show the locations of the two waveforms (i.e., the tips of the corresponding signal vectors) on the below signal space that is spanned by $\{\phi_1(t), \phi_2(t)\}$.**

$S_{ij} = \int_0^{T_b} s_i(t)\phi_j(t)\,dt$, or

find by inspection:

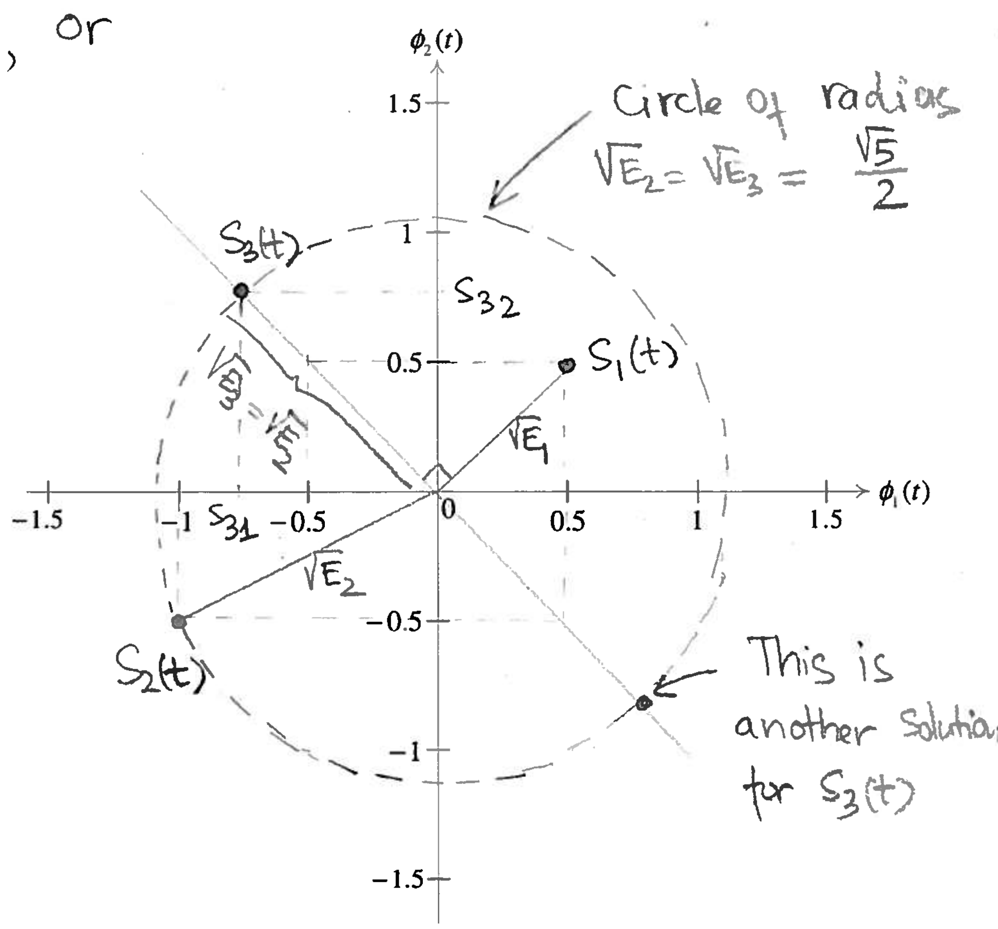
$S_{11} = S_{21} = 0.5$

$S_{21} = -1;\ S_{22} = -0.5$

$-S_{31} = S_{32} = \frac{\sqrt{E_2}}{\sqrt{2}}$

$= \frac{\sqrt{5/4}}{\sqrt{2}} = \frac{\sqrt{5}}{2\sqrt{2}}$

or $S_{31} = -\frac{\sqrt{5}}{2\sqrt{2}}$




[2] (c) **Compute the distance between $s_1(t)$ and $s_2(t)$, either from the signal waveforms or signal space diagram. Also determine whether $s_1(t)$ is orthogonal to $s_2(t)$. Explain your answer.**

$d_{12}^2 = 1^2 + 1.5^2 = 3.25 \Rightarrow d_{12} = \sqrt{3.25} = 1.80$

$S_1(t)$ and $S_2(t)$ are not orthogonal because $\int_0^{T_b} S_1(t)S_2(t)\,dt \neq 0$

[2] (d) **Find and plot the waveform of a signal, called $s_3(t)$, such that (i) $s_3(t)$ has the same energy as that of $s_2(t)$, and (ii) $s_3(t)$ is orthogonal to $s_1(t)$.** *Hint:* The signal space diagram in Part (b) is useful.

See the solution on the figure above.

$S_3(t) = -\frac{\sqrt{5}}{2\sqrt{2}}\phi_1(t) + \frac{\sqrt{5}}{2\sqrt{2}}\phi_2(t)$